MCS CONTENT STANDARDS FOR 1ST GRADE MATHEMATICS

Fluency Expectations or Examples of Culminating Standards

- 1.OA.6: Add and subtract within 20, demonstrating fluency for addition and subtraction within 10. Use strategies such as counting on; making ten (e.g., 8 + 6 = 8 + 2 + 4 = 10 + 4 = 14); decomposing a number leading to a ten (e.g., 13 – 4 = 13 – 3 – 1 = 10 – 1 = 9); using the relationship between addition and subtraction (e.g., knowing that 8 + 4 = 12, one knows 12 – 8 = 4); and creating equivalent but easier or known sums (e.g., adding 6 + 7 by creating the known equivalent 6 + 6 +1 = 12 + 1 = 13).

The following Standards have changes from the 2015-16 MS College- and Career-Readiness Standards:

Significant Changes (ex: change in expecations, new Standards, or removed Standards)
1.MD.3b
1.MD.5

Slight Changes (slight change or clarification in wording)
none

Throughout the 2016 Mississippi College- and Career-Readiness Standards for Mathematics Grades K-5 Standards, the words fluency and fluently will appear in bold, italicized, and underlined font (for example: ***fluently***). With respect to student performance and effective in-class instruction, the expectations for mathematical fluency are explained below:

Fluency is not meant to come at the expense of understanding, but is an outcome of a progression of learning and sufficient thoughtful practice. It is important to provide the conceptual building blocks that develop understanding in tandem with skill along the way to fluency; the roots of this conceptual understanding often extend to one or more grades earlier in the standards than the grade when fluency is finally expected.

Wherever the word ***fluently*** appears in a MS CCR content standard, the word means quickly and accurately. It is important to understand that this is not explicitly tied to assessment purposes, but means more or less the same as when someone is said to be fluent in a foreign language. To be fluent is to flow: Fluent isn’t halting, stumbling, or reversing oneself.

A key aspect of fluency is this sense that it is not something that happens all at once in a single grade but requires attention to student understanding along the way. It is important to ensure that sufficient practice and extra support are provided at each grade to allow all students to meet the standards that call explicitly for fluency.

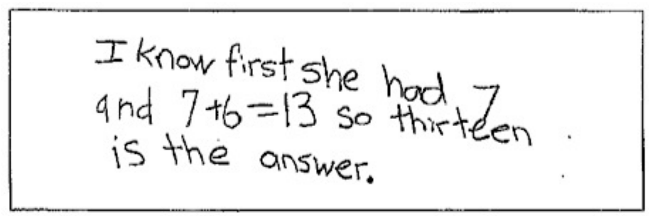
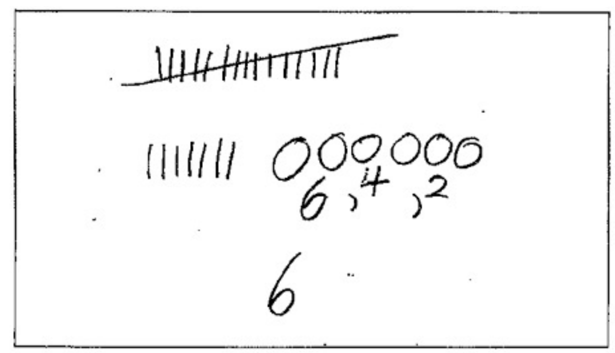
2016 Mississippi College- and Career-Readiness Standards for Mathematics, p. 19

Operations and Algebraic Thinking

Cluster

Represent and solve problems involving addition and subtraction.

Vocabulary: add to, take from, put together/take apart, compare, unknown, sum, difference, less than, more than, equal to

Standard	Clarification
1.OA.1 Use addition and subtraction within 20 to solve word problems involving situations of adding to, taking from, putting together, taking apart, and comparing, with unknowns in all positions, e.g., by using objects, drawings, and equations with a symbol for the unknown number to represent the problem.[2] [2] See Table 1. (*continued on next page*)	This Standard refers directly to Table 1 at the end of this document. Table 1 identifies which addition/subtraction problem types students should already have explored in Kindergarten. By the end of Grade 1, students should have experience with all of the problem types in Table 1; however, they are only expected to "master" those labeled as 1st grade. The complexity of problem types increases in Grade 1 in that they may be more difficult to model directly (a strategy often used by Kindergarteners). Problems in which the "change" or "start" is unknown (see Table 1) are more difficult for children to model and solve because they cannot simply model the action of the story from beginning to end and look at the result. Consider the problem and samples of student work below: <u>Example</u> (Add to, Change Unknown): "Millie's mom gave her 7 stamps from France. Then her grandma gave her some stamps from Egypt. Now Millie has 13 stamps. How many stamps did Millie get from her Grandma?"  Student 1  Student 2 Student 1 recognized the appropriate mathematical relationships needed to solve the problem but lost sight of what the question was asking. She may be accustomed to "the answer" being "the bigger number" (as in an Add to, Result Unknown problem). Student 2 began the problem by using tally marks to model how many stamps Millie had to start and then "counting on" until the total was 13. However, this picture strategy prevented him from seeing how many "new stamps" (from Egypt) that Millie received. He crossed out that initial picture and began again, using tally marks for the 7 stamps that Millie had at first and then switching to circles to represent the stamps that Millie's grandma gave her until he had a total of 13 objects. This enabled him to go back and count up the "new" stamps to see the answer of "6" – Millie's grandma gave her 6 stamps.

1.OA.1 (*cont'd*)
Use addition and subtraction within 20 to solve word problems involving situations of adding to, taking from, putting together, taking apart, and comparing, with unknowns in all positions, e.g., by using objects, drawings, and equations with a symbol for the unknown number to represent the problem.[2]

[2] See Table 1.

The teaching article "Making the Most of Story Problems" (Jacobs and Ambrose, 2009) provides examples of common struggles that students face in solving these types of problems, as well as recommended strategies for helping them move forward:

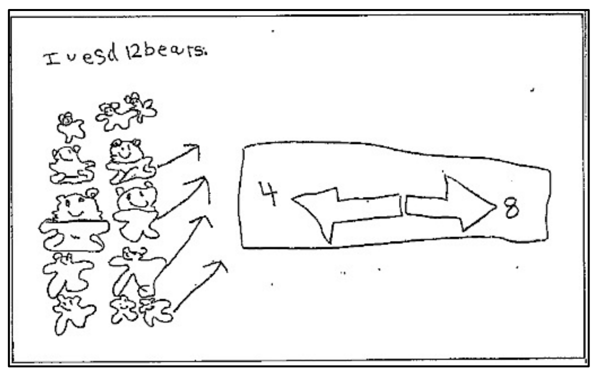
- Jacobs, V. R., & Ambrose, R. C. (2008, December). Making the most of story problems. *Teaching Children Mathematics*, 260-266.

"Compare" problems are introduced for the first time in Grade 1. The most powerful modeling strategy for this type of problem is a "matching" strategy: students use blocks or counters to model the amounts in each set and then see "How many more?" or "How many less?" one set has than the other. This builds directly off of Standards **K.CC.4** and **K.CC.6**.

TEACHER NOTE: The purpose of this Standard is for First Graders to model the mathematics of the problem (with manipulatives, drawings, etc.) *first*. After representing the problem, the teacher may then guide discussions on how to represent what the students modeled/drew with mathematics (equations). This follows the concrete → pictorial → abstract progression that promotes long-term understanding. When modeling strategies are not effective, this is an opportunity for students to explore using counting strategies, number relationships, and other mathematical relationships to solve problems. (See **1.OA.3**, **1.OA.4**, and **1.OA.6**.)

Students should have experience with using manipulatives and pictures to solve addition/subtraction problems in Kindergarten (**K.OA.1**, **K.OA.2**). Below is an example of a "Take from, Result Unknown" problem (a Kindergarten problem type) and examples of how First Graders used pictures to represent how they solved it:

"There are 12 ants on the ground. Then 4 ants go down their ant hole. How many ants are still on the ground?"

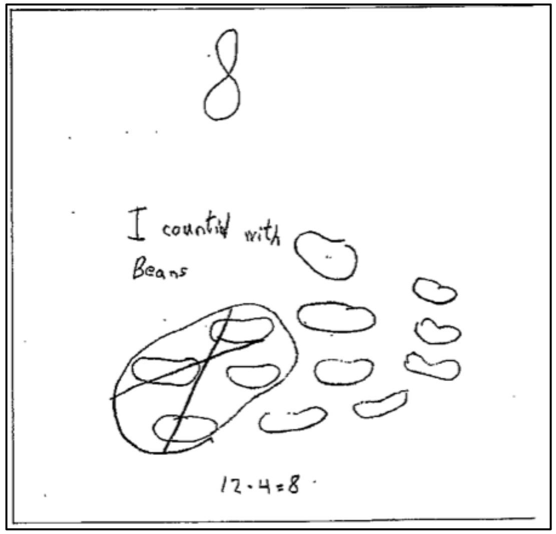


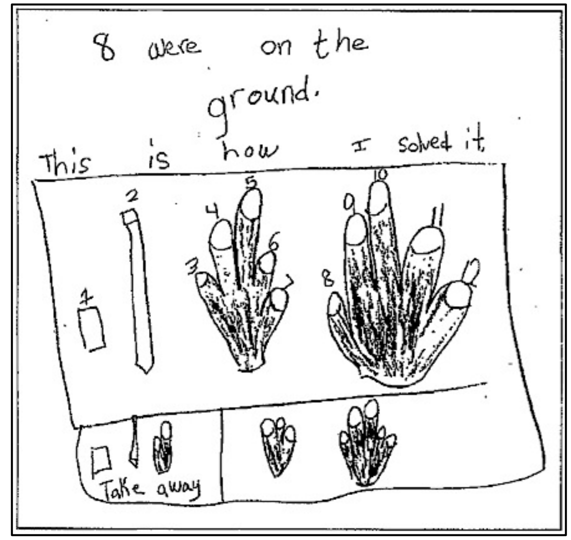
(*continued on next page*)

1.OA.1 (*cont'd*)
Use addition and subtraction within 20 to solve word problems involving situations of adding to, taking from, putting together, taking apart, and comparing, with unknowns in all positions, e.g., by using objects, drawings, and equations with a symbol for the unknown number to represent the problem.[2]

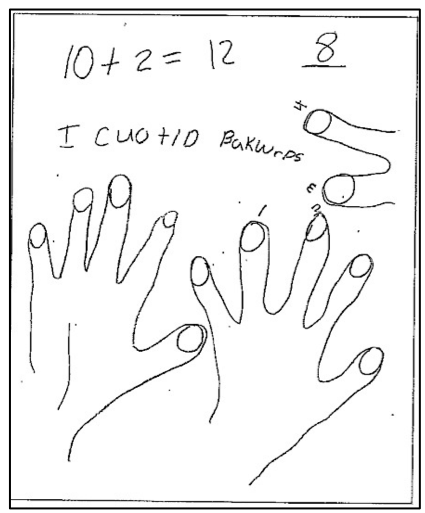
[2] See Table 1.

Examples of other First Graders' drawings for the "ants on the ground" problem on the previous page:





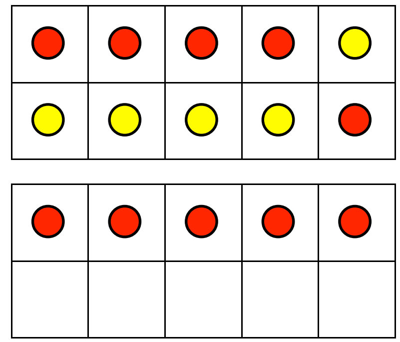
First Graders also build on the methods they used in Kindergarten (counting) to add and subtract within this larger range. First Grade students should explore problems that promote strategies such as counting on, counting back, making ten, and doubles +/- 1 or +/- 2 to solve problems. Below is an example of how a student represented his counting strategy to the previous "ants on the ground" problem:



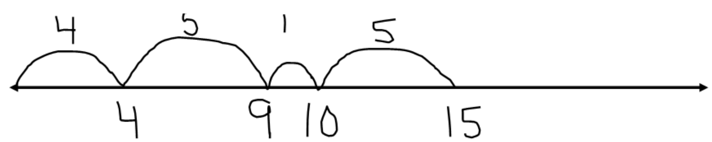
1.OA.2
Solve word problems that call for addition of three whole numbers whose sum is less than or equal to 20, e.g., by using objects, drawings, and equations with a symbol for the unknown number to represent the problem.

First Grade students solve multi-step word problems by adding (joining) three numbers whose sum is less than or equal to 20, using a variety of mathematical representations. After modeling the problem first, students may write an equation or number sentence that best describes their model/strategy.

<u>Example</u>:
Mrs. Smith is counting cookies. She has 4 oatmeal raisin cookies, 5 chocolate chip cookies, and 6 gingerbread cookies. How many cookies does Mrs. Smith have?



<u>Student A</u> <talking>: I used a double ten frame. I put down 4 counters for oatmeal raisin cookies. Then, I used 5 counters for chocolate chip. I changed colors to help me remember which were which. Then, I put 6 more down for gingerbread. I filled a whole ten frame and five more. 10 and 5 are 15. So, she has 15 cookies.



<u>Student B</u> <talking>: I used a number line. First I jumped to 4 for oatmeal raisin cookies. Then I jumped 5 more for chocolate chip and landed on 9. I know 9 plus 1 is 10, so I did that first. That's 1 gingerbread cookie. I needed to jump 5 more to finish the gingerbread ones. So I did that and landed at 15. I used up all of the cookies, so I know I'm done. So, Mrs. Smith has 15 cookies.

Cluster

Understand and apply properties of operations and the relationship between addition and subtraction.

Vocabulary: add, sum, subtract, difference, order, addend

1.OA.3
Apply properties of operations as strategies to add and subtract.[3]

Examples: If 8 + 3 = 11 is known, then 3 + 8 = 11 is also known. (Commutative property of addition). To add 2 + 6 + 4, the second two numbers can be added to make a ten, so 2 + 6 + 4 = 2 + 10 = 12 (Associative property of addition).

[3] Students need not use formal terms for these properties.

The purpose of this Standard is <u>not</u> to use direct or explicit instruction to teach First Graders about properties of operations. The purpose is to provide First Graders with numerous experiences in modeling number relationships and addition/subtraction scenarios that they begin to recognize these properties and relationships for themselves. Teachers may use questioning techniques to draw students' attention to these properties or to help students articulate their thinking.

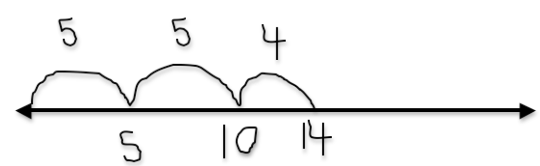
While the Standard says that "students need not use formal terms for these properties," that does not mean students should be taught incorrect or non-mathematical terms in their place. For example, teachers should not describe The Commutative Property of Addition as "The Old Switcharoo Property" instead.

Students should have informal experience with The Commutative Property of Addition from exploring "Put Together/Take Apart, Both Addends Unknown" problems in Kindergarten. The book *Quack and Count* tells the story of 7 ducklings on an adventure. The pictures provide excellent opportunities to count the ducks in different arrangements. (Ex: How many ducks are in the water? How many ducks are not in the water?) The teacher can then record the number relationships (Ex: 7 = 3 + 4) on the board, as the class reads through the story. There are always 7 ducklings, no matter how they are arranged.

Example of how students might use The Associative Property of Addition at this grade level:

<u>Example</u>: Dylan is counting his racecar collection. He has 5 blue cars, 4 green cars, and 5 red cars. How many cars are in Dylan's collection?

<u>Student A</u><talking>: First I jumped to 5 for the blue cars. I saw there was another 5 – 5 red cars – and I know 5 and 5 is 10. So, I did that next. Then I jumped 4 more for the green cars and landed on 14. Dylan has 14 cars.



<u>Student B</u> <talking>: I wrote a number sentence. I put 5 and 5 together first because I like working with tens. Then I just had 4 more, and 10 plus 4 is 14. So, Dylan had 14 racecars.

5 + 4 + 5 = □

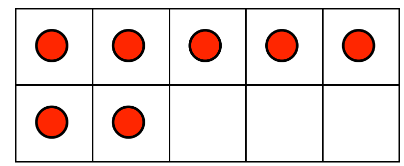
10 + 4 = 14

<table>
<tr><td>1.OA.4
Understand subtraction as an unknown-addend problem.

For example, subtract 10 – 8 by finding the number that makes 10 when added to 8.</td><td>The purpose of this Standard is to help First Graders understand that you can subtract to find the difference between two amounts, or you can add on (or count on) to determine the difference, as well. Many people (young and old), actually use addition to solve what might typically be described as “subtraction” problems.

This Standard builds on K.OA.4 in which students learn how to complete a ten, given a single digit number. It also builds on K.OA.5 in which students fluently add and subtract within 5.

<u>Example</u> (Take from, Change Unknown):
Kimberly had 10 crayons on her desk. Some crayons rolled under her desk. Then Kimberly had 7 crayons. How many crayons rolled under Kimberly’s desk?

<u>Student</u>: She had 10. Now she has 7. I put 7 counters on my ten frame. I need 3 more to make 10. So, 3 crayons must’ve rolled under the desk.


<u>Example</u> (Take from, Change Unknown):
Marcus had 15 dollars. He spent some money on comic books. Now he has 9 dollars. How much money did Marcus spend?

<u>Student</u>: Well, 9 plus 1 is 10 <holds up one finger>, and then 11, 12, 13, <holding up one finger at a time as he counts>, 14, 15. That’s 6. So, he spent 6 dollars. 9 and 6 more puts him back at 15.</td></tr>
<tr><td colspan="2">Cluster</td></tr>
<tr><td colspan="2">Add and subtract within 20.</td></tr>
<tr><td colspan="2">Vocabulary: addition, subtraction, counting on, counting back</td></tr>
<tr><td>1.OA.5
Relate counting to addition and subtraction (e.g., by counting on 2 to add 2).

(continued on next page)</td><td>When solving addition and subtraction problems to 20, First Graders often use counting strategies (ex: counting all, counting on, counting back) before fully developing the strategy of using 10 as a benchmark number. Over time, it is important for students to recognizing that “counting on two more” will give them the same result as “adding two.” (Ex: 8… 9, 10 and 8 + 2 = 10)

It is important to provide students with tasks that help them build on this knowledge and also promote composing and decomposing numbers to solve problems, particularly since counting becomes a less efficient strategy when working with larger numbers. Students often find 10 or multiples of 10 to be helpful benchmarks when composing and decomposing larger numbers.</td></tr>
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<tr>
<td>1.OA.5 (cont'd)
Relate counting to addition and subtraction (e.g., by counting on 2 to add 2).</td>
<td>
Examples of Counting Strategies
Counting All: Students count each set of objects starting at 1 to determine the resulting amount. At this stage, they depend on starting back at 1 to count "how many."
Counting On & Counting Back: Students hold a "start number" in their head and count on/back from that number, rather than starting at 1. These indicate a more sophisticated understanding of numbers than "counting all."
Example (Add to, Result Unknown):
Kasey had 15 stickers on her chart. She got two more stickers for good behavior. How many stickers does Kasey have now?
<table>
<tr><th>Counting All</th><th>Counting On</th></tr>
<tr><td>The student counts out 15 counters. The student counts out 2 more counters. The student then puts the two sets together and counts all of the counters starting at 1 (1, 2, 3, 4, 5,… 14, 15, 16, 17) to find the total amount of 17.</td><td>Starting at 15, the student holds up one finger and says, "16," then holds up another finger and says, "17. She has 17 stickers." The student counted on 2 to find that 15 + 2 = 17.</td></tr>
</table>
Example (Take from, Result Unknown):
Tristan had 12 pieces of candy. He ate 3 pieces. How many pieces of candy does Tristan have now?
<table>
<tr><th>Counting All</th><th>Counting Back</th></tr>
<tr><td>The student counts out 12 counters. The student then moves 3 counters (counting "1, 2, 3…") from that set to the side. The student then counts each of the remaining counters, starting again at 1 ("1, 2, 3, 4, 5, 6, 7, 8, 9") to determine the final result of 9.</td><td>Starting at 12, the student holds up one finger and says, "11" (to indicate subtracting 1 from 12); then he holds up a second finger and says, "10," and then a third finger and says, "9. He has 9 pieces left." The student counted back 3 from 12 to find that 12 – 3 = 9.</td></tr>
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<tr>
<td>1.OA.6
Add and subtract within 20, demonstrating fluency for addition and subtraction within 10. Use strategies such as counting on; making ten (e.g., 8 + 6 = 8 + 2 + 4 = 10 + 4 = 14); decomposing a</td>
<td>"The word fluent is used in the Standards to mean "fast and accurate." Fluency in each grade involves a mixture of just knowing some answers, knowing some answers from patterns (e.g., "adding 0 yields the same number"), and knowing some answers from the use of strategies. It is important to push sensitively and encouragingly toward fluency of the designated numbers at each grade level, recognizing that fluency will be a mixture of these kinds of thinking which may differ across students." (Progressions for the CCSSM (Draft): K, Cardinality; K-5, Operations and Algebraic Thinking, May 2011, p. 18)</td>
</tr>
</table>

number leading to a ten (e.g., 13 – 4 = 13 – 3 – 1 = 10 – 1 = 9); using the relationship between addition and subtraction (e.g., knowing that 8 + 4 = 12, one knows 12 – 8 = 4); and creating equivalent but easier or known sums (e.g., adding 6 + 7 by creating the known equivalent 6 + 6 +1 = 12 + 1 = 13).

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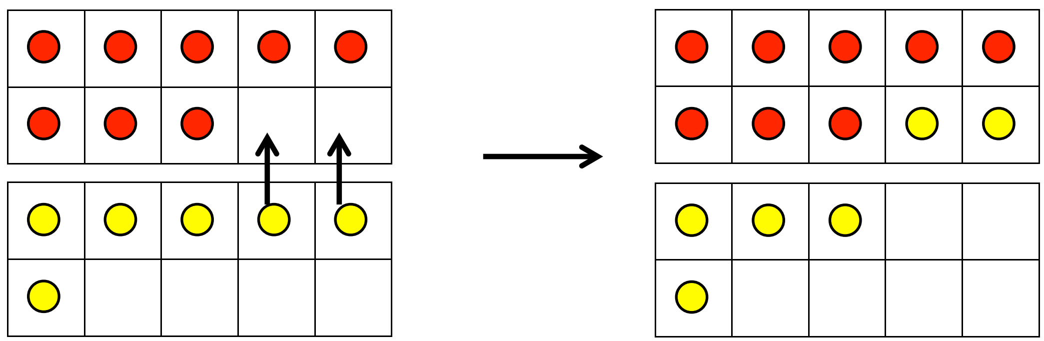
Research indicates that teachers can best support students' memory of the sums of two one-digit numbers through varied experiences including making 10, breaking numbers apart, and working on mental strategies. These strategies replace the use of repetitive timed tests in which students try to memorize operations as if there were not any relationships among the various facts. (Fosnot & Dolk, 2001)

Students who are comfortable with number relationships can often reason through 6 + 7 as 6 + 6 + 1, which is 12 + 1 = 13, quickly and efficiently. Students who understand how to use this type of strategy ("doubles + 1") are less likely to make mistakes than students who have tried to memorize facts without any relationships or understanding to support those facts.

Ten frames and number lines can be powerful models to help students make sense of the strategies described in this Standard.

<u>Example</u> (Put Together/Take Apart, Total Unknown):
8 red apples and 6 yellow apples are on the tree. How many apples are on the tree?

<u>Student</u> <talking>: I used a double ten frame. I put 8 counters on the first frame for the 8 red apples. Then I put 6 yellow counters on the second frame for the yellow apples I saw that if I moved 2 yellow counters up, I could make a full ten. So, I did that, and then I had 10 and 3 and 1, which is 10 and 4. That's 14. So, there are 14 apples on the tree.



This strategy of "making a ten" can then be represented symbolically:

$$8 + 6 =$$

$$2 \quad 4$$

$$10 + 4 = 14$$

1.OA.6 (*cont'd*)
Add and subtract within 20, demonstrating fluency for addition and subtraction within 10. Use strategies such as counting on; making ten (e.g., 8 + 6 = 8 + 2 + 4 = 10 + 4 = 14); decomposing a number leading to a ten (e.g., 13 – 4 = 13 – 3 – 1 = 10 – 1 = 9); using the relationship between addition and subtraction (e.g., knowing that 8 + 4 = 12, one knows 12 – 8 = 4); and creating equivalent but easier or known sums (e.g., adding 6 + 7 by creating the known equivalent 6 + 6 +1 = 12 + 1 = 13).

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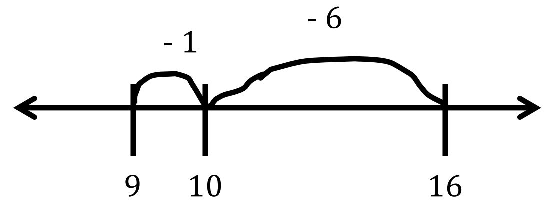
Example (Compare, Smaller Unknown):
Miguel had 16 markers. Evan had 7 fewer markers than Miguel did. How many markers did Evan have?



Student <talking>: I used a number line. I started at 16 for Miguel's markers. Evan had 7 fewer. First I jumped back 6 because that was easier for me to do. Then I jumped 1 more. I landed on 9. So Evan had 9 markers.

The strategy above may be referred to as "backing down through ten."

This Standard lays the foundation for strategies students will use in Grade 2 with larger numbers:

Example of "Making Ten" in Grade 1

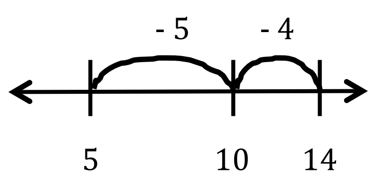
9 + 7 =
1 6
10 + 6 = 16

Example of "Making Tens/Friendly Numbers" in Grade 2

29 + 47 =
1 46
30 + 46 = 76

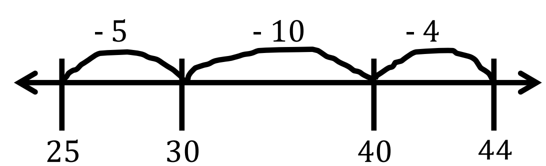
Example of "Backing Down Through 10" in Grade 1

14 - 9 =
14 - 4 = 10
10 - 5 = 5



Application of "Backing Down Through 10" in Grade 2

44 - 19 =
44 - 4 = 40
40 - 10 = 30
30 - 5 = 25



	<u>TEACHER NOTE</u>: There has been increasing research within the past several years describing the negative impact that timed tests and drills have on students. The overall consensus is that timed tests and flash card drills are not effective means to help students learn “facts” with long-term success. Students who are strong memorizers may have success with these assessments, but that does not mean that they <u>understand</u> the number relationships. For more information on the negative impact of timed tests and alternative assessment strategies that promote number sense and long-term success, see • Boaler, J. (2014, April). Research suggests that timed tests cause math anxiety. *Teaching Children Mathematics*, *20*(8), 469-474. • Kling, G., & Bay-Williams, J. M. (2014, April). Assessing basic fact fluency. *Teaching Children Mathematics*, *20*(8), 488-497.
Cluster	
Work with addition and subtraction equations.	
Vocabulary: equation, equal, true, false	
1.OA.7 Understand the meaning of the equal sign, and determine if equations involving addition and subtraction are true or false. *For example, which of the following equations are true and which are false? 6 = 6, 7 = 8 - 1, 5 + 2 = 2 + 5, 4 + 1 = 5 + 2*	The most common misinterpretation of the equal sign is that it means, “The answer comes next.” Teachers can help combat this misinterpretation by writing equations in different arrangements: 2 + 3 = 5 *and* 5 = 2 + 3. Classroom discussions about the equal sign representing “balance” or “the same value” are also important. A helpful teaching article on identifying and addressing children’s conceptions of the equal sign is • Witherspoon, M. L. (1999, March). And the answer is…Symbolic literacy. *Teaching Children Mathematics*, 396-399.
1.OA.8 Determine the unknown whole number in an addition or subtraction equation relating three whole numbers. *For example, determine the unknown number that makes the equation true in each of the equations 8 + ? = 11, 5 = □ - 3, 6 + 6 = □.*	This Standard works along with Standards **1.OA.4** and **1.OA.6**. <u>Example</u>: 5 - □ = 3 <u>Student A</u>: I thought about what goes with 3 to make 5. I know that 3 and 2 is 5. So, the answer is 2. <u>Student B</u>: Five, four, …(*holding up 5 fingers and folding one under for each count*). Three. I have 3 fingers left up and 2 fingers down. So, it’s 2. <u>Student C</u>: We ended with 3. Four, five… (*holding up 1 finger for each count*). Two! 2 goes in the box. 5 – 2 = 3.

Number and Operations in Base Ten

Cluster

Extend the counting sequence.

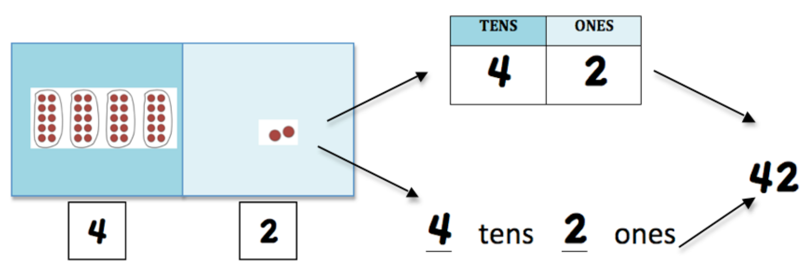
Vocabulary: tens, ones, hundred

1.NBT.1 Count to 120, starting at any number less than 120. In this range, read and write numerals and represent a number of objects with a written numeral.	This Standard builds off of **K.CC.1** and **K.CC.2** in which students count to 100 by tens and ones and then count forward within 100 from a number less than 100. First Grade students rote count forward to 120 by counting on from any number less than 120. Counting forward is more challenging for students than counting from 1 because they must pick up "in the middle" (so to speak) of the counting sequence and recognize what comes next. This requires a deeper understanding of our number system. Our system of writing and naming numbers is simply a "convention" that we can help our students learn. It is common for them to "invert" place values, based on how we pronounce the number names. For example, a student may write "17" and mean "71." This provides the teacher with an opportunity to discuss place value: "Let's look at this: Did you mean seventeen or seventy-one? How many tens and how many ones does 17 have? How many tens and how many ones does 71 have? How would write both of those numbers?"

Cluster

Understand place value.

Vocabulary: tens, ones, bundle, leftovers/singles, groups, compare, greater than, less than, equal to

1.NBT.2 Understand that the two digits of a two-digit number represent amounts of tens and ones. Understand the following as special cases: a. 10 can be thought of as a bundle of ten ones called a "ten." b. The numbers from 11 to 19 are composed of a ten and one, two, three, four, five, six, seven, eight, or nine ones. c. The numbers 10, 20, 30, 40, 50, 60, 70, 80, 90 refer to one, two, three, four, five, six, seven, eight, or nine tens (and 0 ones).	First Grade students are introduced to the idea that a bundle of ten ones is called "a ten." This is known as "unitizing tens." When First Grade students unitize a group of ten ones as a whole unit ("a ten"), they are able to count groups as though they were individual objects. This is a monumental shift in thinking and can often be challenging for young children to consider a group of something as "one" when all previous experiences have been counting single objects. This is the foundation of the place value system and requires time and rich experiences with concrete manipulatives to develop. 

1.NBT.2 (*cont'd*)
Understand that the two digits of a two-digit number represent amounts of tens and ones. Understand the following as special cases:

a. 10 can be thought of as a bundle of ten ones called a "ten."

b. The numbers from 11 to 19 are composed of a ten and one, two, three, four, five, six, seven, eight or nine ones.

c. The numbers 10, 20, 30, 40, 50, 60, 70, 80, 90 refer to one, two, three, four, five, six, seven, eight, or nine tens (and 0 ones).

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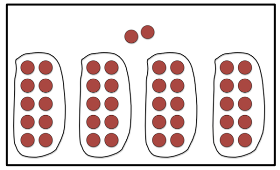
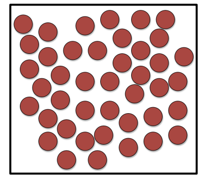
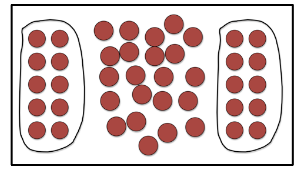
A student's ability to conserve number is an important aspect of this standard. It is not obvious to young children that 42 cubes is the same amount as 4 groups of ten and 2 leftovers. It is also not obvious that 42 could also be composed of 2 groups of 10 and 22 leftovers. Therefore, first graders require ample time grouping proportional objects (e.g., cubes, beans, beads, ten-frames) to make groups of ten, rather than using only pre-grouped materials (e.g., Base 10 Blocks, pre-made bean sticks) that have to be "traded" or are non-proportional (e.g., money).

Example: 42 counters can be grouped many different ways and still remain a total of 42:


4 groups of 10 and 2 more


42 single counters

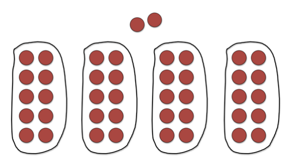

2 groups of ten and 22 singles

"We want children to construct the idea that all of these are the same and that the sameness is clearly evident by virtue of the groupings of ten. Groupings by tens is not just a rule that is followed but that any grouping by tens, including all or some of the singles, can help tell how many." (Van de Walle & Lovin, p. 124)

As children build this understanding of grouping, they move through several stages: Counting By Ones; Counting by Groups & Singles; and Counting by Tens and Ones.

Counting By Ones: At first, even though First Graders will have grouped objects into tens and leftovers, they rely on counting all of the individual cubes by ones to determine the final amount. It is seen as the only way to determine how many.

Example:



Teacher: How many counters do you have?
Student: 1, 2, 3, 4, 5, 6, 7, … 41, 42. I have 42 counters.

1.NBT.2 (*cont'd*)
Understand that the two digits of a two-digit number represent amounts of tens and ones. Understand the following as special cases:

a. 10 can be thought of as a bundle of ten ones called a "ten."

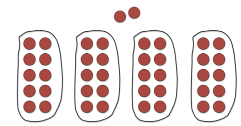
b. The numbers from 11 to 19 are composed of a ten and one, two, three, four, five, six, seven, eight, or nine ones.

c. The numbers 10, 20, 30, 40, 50, 60, 70, 80, 90 refer to one, two, three, four, five, six, seven, eight, or nine tens (and 0 ones).

(*continued on next page*)

<u>Counting By Groups and Singles</u>: While students are able to group objects into collections of ten and now tell how many groups of tens and leftovers there are, they still rely on counting by ones to determine the final amount. They are unable to use the groups and leftovers to determine how many.

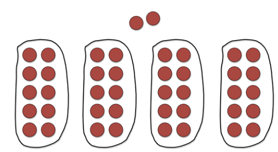
<u>Example</u>:



Teacher: How many counters do you have?
Student: I have 4 groups of ten and 2 leftovers.
Teacher: Does that help you know how many you have?
Student: Let me see. 1, 2, 3, 4, 5, 6, 7, … 41, 42. I have 42 counters.

<u>Counting by Tens & Ones</u>: Students are able to group objects into ten and ones, tell how many groups and leftovers there are, and now use that information to tell how many. Occasionally, as this stage is becoming fully developed, first graders rely on counting by ones to "really" know how many there are, even though they may have just counted the total by groups and leftovers.

Example:



Teacher: How many counters do you have?
Student: I have 4 groups of ten and 2 leftovers.
Teacher: Does that help you know how many you have?
Student: Yes. That means that I have 42 counters.
Teacher: Are you sure?
Student: Um. Let me count just to make sure… 1, 2, 3, … 41, 42. Yes. I was right. There are 42 counters.

1.NBT.2 (*cont'd*)
Understand that the two digits of a two-digit number represent amounts of tens and ones.
Understand the following as special cases:

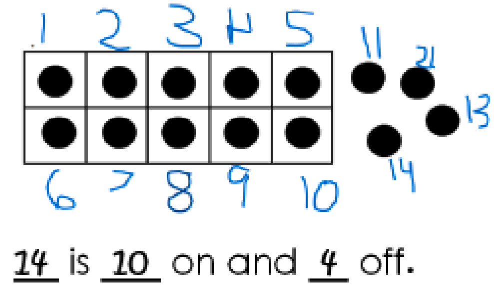
a. 10 can be thought of as a bundle of ten ones called a "ten."

b. The numbers from 11 to 19 are composed of a ten and one, two, three, four, five, six, seven, eight, or nine ones.

c. The numbers 10, 20, 30, 40, 50, 60, 70, 80, 90 refer to one, two, three, four, five, six, seven, eight, or nine tens (and 0 ones).

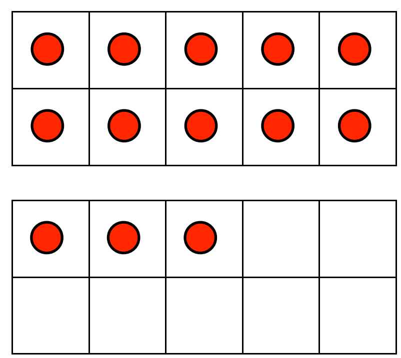
This Standard also builds off of **K.NBT.1** in which students decomposed numbers 11 – 19 into ten and "some more ones."

Ten Frames, Double Ten Frames, and Rekenreks can be powerful models for exploring these relationships. Students can use single Ten Frames to describe 11 – 19 as some "on the ten frame" and some "off of the ten frame." For example,



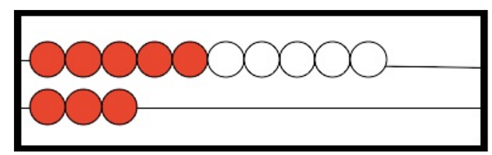
ALL	On	Off
14	10	4

Working with a Double Ten Frame or Rekenrek, students can still explore the relationships between the numbers:



Double Ten Frames

13 = 10 + 3



Rekenrek

13 = 10 + 3

First Grade teachers can build on these models to model and discuss place value (how many groups of ten, how many ones) with bigger numbers (ex: 24, 33, etc.).

1.NBT.3
Compare two two-digit numbers based on meanings of the tens and ones digits, recording the results of comparisons with the symbols >, =, and <.

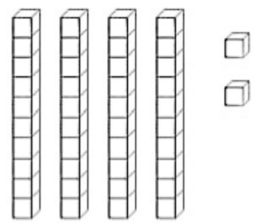
Key topics within this Standard:

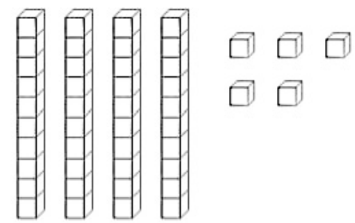
- Comparing numbers should begin by using models (ex: Ten Frames, Base 10 Blocks) first before numbers are compared as "digits alone."
- Students should use words first to compare numbers (more than, less than, the same as) before using symbols.
- Language such as, "The alligator eats the bigger number" is not mathematical and should be avoided. The symbols for greater than and less than are designed to be proportional so that the "bigger side" (more space) is next to the bigger number and the "smaller side" (less space) is next to the smaller number.

Students may also use "counting strategies" to compare numbers – recognizing that if one number comes before the other in the traditional counting sequence, it is less than the other.

Example: Which is bigger? 42 or 45?

Student A: I used Base 10 blocks. I used 4 longs and 2 units for 42 and 4 longs and 5 units for 45.





Student A (cont'd): They both have 4 longs – 4 tens – but 45 has more units – more ones. So, 45 is more than 42.

Teacher: How could we use our symbols to describe that?

Student A <writes>: 45 > 42

Student B: 42 is less than 45. I know this because when I count up, I say 42 before I say 45.

Teacher: How could we use our symbols to describe that?

Student B <writes>: 42 < 45.

Cluster

Use place value understanding and properties of operations to add and subtract.

Vocabulary: hundred, tens, ones, compose, place value, strategy

1.NBT.4
Add within 100, including adding a two-digit number and a one-digit number, and adding a two digit number and a multiple of 10, using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method and explain the reasoning used. Understand that in adding two-digit numbers, one adds tens and tens, ones and ones; and sometimes it is necessary to compose a ten.

(*continued on next page*)

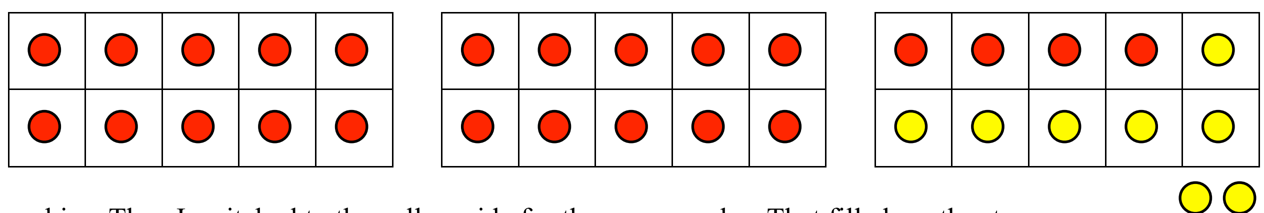
This Standard emphasizes the importance of place value, number relationships, and mathematical reasoning. **Students are not expected to be fluent with the standard algorithms for addition and subtraction until the end of Grade 4** (**4.NBT.4**). However, students can and should have experience in using the traditional algorithm to add and subtract, as it is often efficient.

The word "algorithm" refers to a procedure or a series of steps that when followed will produce a correct solution. Students should be able to explain how they used the traditional algorithm based on understanding of place value and number (**2.NBT.9**). "Explanations" such as "I followed the steps." or "More on the floor? Go next door!" are not mathematical explanations and do not demonstrate deep understanding.

Students should be able to explain their thinking (such as why they chose a particular model or method to solve the problem) and use number sense or estimation to make sure that their answer is reasonable.

Example (Put Together/Take Apart, Total Unknown):
24 red apples and 8 green apples are on the table. How many apples are on the table?

Student A <talking>: I used Ten Frames. There were 24 red apples. That was two full Ten Frames and then four



more chips. Then I switched to the yellow side for the green apples. That filled another ten, and I had 2 chips left. So that's 10, 20, 30, 31, 32. So, 24 + 8 = 32.

This strategy is one that focuses on place value and on composing tens. It can be represented symbolically as

$$24 + 8 =$$

$$6 \quad 2$$

$$30 + 2 = 32$$

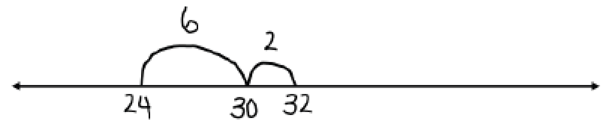
1.NBT.4 (*cont'd*)
Add within 100, including adding a two-digit number and a one-digit number, and adding a two digit number and a multiple of 10, using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method and explain the reasoning used. Understand that in adding two-digit numbers, one adds tens and tens, ones and ones; and sometimes it is necessary to compose a ten.

Example (Put Together/Take Apart, Total Unknown):
24 red apples and 8 green apples are on the table. How many apples are on the table?

Student B:
I used an empty number line. I started at 24 because that's how many red apples there were. Next I jumped 6 more to 30 because that was a friendly jump for me. So, that was 6 green apples, and now I have to jump 2 more to finish the 8 green apples. I landed on 32. So, there are 32 apples on the table.

24 + 6 = 30

30 + 2 = 32



Student C:
It's easier for me to work with 10 than 8, so I thought, "Suppose it was 10 green apples instead." 24 and 10 more is 34. But, since I added 2 in at the beginning, I had to take them out again. 34 minus 2 is 32. So, there are 32 apples on the table.

8 + 2 = 10

24 + 10 = 34

34 – 2 = 32

Example (Compare, Difference Unknown):
There were 63 people at the zoo. There were 83 people at the park. How many more people were at the park than at the zoo?

Student A <talking>: I used a hundreds chart. I started at 63 because that's how many people were at the zoo. Then I jumped down one row to 73. When you jump down a row, that's like adding 10. Then I jumped another row down – another 10 – and landed on 83. 10 and 10 is 20. So, there were 20 more people at the park.

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

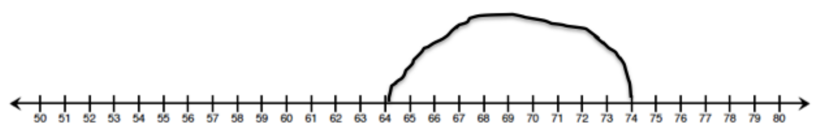
1.NBT.5
Given a two-digit number, mentally find 10 more or 10 less than the number, without having to count; explain the reasoning used.

First Graders build on their counting by tens work in Kindergarten by mentally adding ten more and ten less than any number less than 100. First Graders are not expected to compute differences of two-digit numbers other than multiples of ten. (See **1.NBT.4** and **1.NBT.6**.)

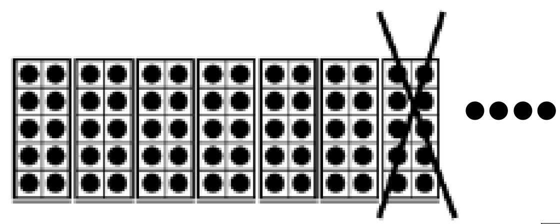
Ten Frames, Hundreds Charts, and the number line are powerful tools that students can use to model adding and subtracting tens. The goal is for students to work with these representations enough so that they internalize the relationships and can then solve these types of problems mentally.

Example: There are 74 birds in the park. 10 birds fly away. How many birds are in the park now?

Student A <talking>: I used a number line. I started at 74 because that's how many birds were in the park. Then I jumped back 10 because 10 birds flew away. I landed on 64. So, there are 64 birds left in the park.

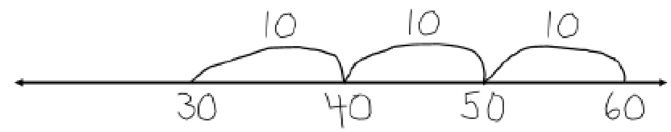
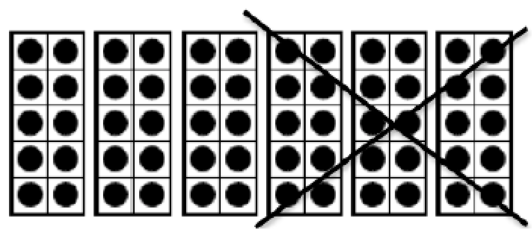


Student B <talking>: I got out 7 full ten frames and 4 counters. Since 10 birds flew away, I took one of the ten frames away. That left 6 ten frames and 4 left over. So, there are 64 birds left in the park.



Student C
I thought about the hundreds chart in my head. I know that if I jump back a row, that's 10 less. If I start at 74 and jump back a row, I'll land on 64. So there are 64 birds in the park.

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

1.NBT.6 Subtract multiples of 10 in the range 10 – 90 from multiples of 10 in the range 10 – 90 (positive or zero differences), using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method and explain the reasoning used.	First Grade students use concrete models, drawings and place value strategies to subtract multiples of 10 from decade numbers (e.g., 30, 40, 50). They often use similar strategies to those discussed in **1.OA.4**. Example (Take from, Result Unknown): There are 60 students in the gym. 30 students leave. How many students are still in the gym? Student A <talking>: I used a number line. I started at 60 because that's how many students were in the gym. I moved back 3 jumps of 10 to show the 30 students who left. I did that because it's easier for me to jump by tens. I finally landed on 30. So there are 30 students left.  Student B <talking>: I used ten frames. I got out 6 full ten frames - that's 6 groups of 10, which is 60. I took away three ten frames because 30 students left the gym. I had 3 ten frames left, so there are 30 students left.  $\boxed{60 - 30 = 30}$ Student C <talking>: I thought, "30 and what makes 60?" I know 3 and 3 is 6. So, 3 tens and 3 tens would be 6 tens, which is 60. So, there must be 3 tens or 30 left. So 30 students are still in the gym. $\boxed{30 + 30 = 60}$

Measurement and Data

Cluster

Measure lengths indirectly and by iterating unit lengths.

Vocabulary: measure, length, height, more than, less than, longer than, shorter than, order, first, second, third, gap, overlap, units, about, a little more than, a little less than

1.MD.1
Order three objects by length; compare the lengths of two objects indirectly by using a third object.

First Grade students continue to use direct comparison (**K.MD.2**) to compare lengths. *Direct* comparison means that students compare the amount of an attribute in two objects without using formal measurement units.

Example: Who is taller?

Student: Let's stand back to back and compare our heights. Look! I'm taller!

Example: Find at least 3 objects in the classroom that are the about the same length as, longer than, and shorter than a pencil.

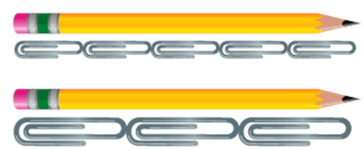
In Grade 1, measurement experiences are extended to include *indirect comparisons* – using a third object as a comparison tool for length. For example, if we know that Aleisha is taller than Barbara and that Barbara is taller than Callie, then we know that Aleisha is taller than Callie, even if Aleisha and Callie never stand back to back. This concept is referred to as the "transitivity principle*" for indirect measurement. (Students do not need to know or use the term "transitivity principle," but they should have experiences in comparing measurements this way.)

Example: The snake handler is trying to put the snakes in order from shortest to longest. Here are the three snakes (3 strips of paper of different length and color). Can you help the snake handler put them in order?

Student: I'm going to start by putting the snakes next to each other. It helps if you line them up so they all start at the same place because otherwise it's hard to see for sure. Okay… the blue snake is the shortest of all of them. The red snake is really long. The green snake is longer than the blue snake, but not as long as the red one. So, I think she should put them blue, red, green.

Snakes		
Shortest	⟷	Longest
blue	green	red

The Transitivity Principle ("transitivity"): If the length of object A is greater than the length of object B, and the length of object B is greater than the length of object C, then the length of object A is greater than the length of object C. This principle applies to measurement of other quantities as well.

1.MD.2 Express the length of an object as a whole number of length units, by laying multiple copies of a shorter object (the length unit) end to end; understand that the length measurement of an object is the number of same-size length units that span it with no gaps or overlaps. *Limit to contexts where the object being measured is spanned by a whole number of length units with no gaps or overlaps.*	First Graders use objects to measure items to help students focus on the attribute being measured. Objects also lend themselves to future discussions regarding the need for a standard unit. First Grade students use multiple copies of one object to measure the length of a larger object. They learn to lay physical units such as manipulatives with side lengths of 1 cm or 1 inch end-to-end and count the number of units used to measure a length. Through numerous experiences and careful questioning by the teacher, students will recognize the importance of careful measuring so that there are not any gaps (distances left unmeasured) or overlaps (distances measured more than once) in order to get an accurate measurement. The concepts and skills of measurement in Grade 2 (**2.MD.2**) build directly on this foundation. Example: How long is the pencil, using paper clips to measure? Student: I lined up my paper clips under the pencil. I made sure that I didn't leave any spaces or overlap them. The pencil is about 5 little paperclips long. I tried it with the big paperclips, too. It's about 3 big paperclips long.  When students use different sized units to measure the same object, they learn that the sizes of the units must be considered, rather than relying solely on the amount of objects counted. Teacher: (Following up on the previous discussion) So, the first time you measured it, you said the pencil was 5 little paperclips long. And then when you measured again, you said it was 3 big paperclips long. Isn't 3 less than 5? What happened? Did the pencil shrink? Student: Umm…. No, it was the same pencil; it didn't change. The little paperclips are umm, … little. It'll take more of them to measure the pencil than it would with big paperclips. 5 is more than 3, but you have to look at it carefully. You've got to pay attention to what you're using. If you're using something small, it'll take more of those than if you used something bigger to measure. ***TEACHER NOTE***: When using manipulatives, such as centimeter cubes or square tiles, to measure length, discussion should focus on the length of the edges or sides of the unit being used as a measure of distance. For example, a square tile itself represents a unit of area (one square inch). But *the length of the side of a square tile* represents a length measurement of one inch.

Cluster

Tell and write time with respect to a clock and a calendar.

Vocabulary: time, hour, half hour, minutes, hour hand, minute hand, about, o'clock, past, until, "six thirty"

1.MD.3

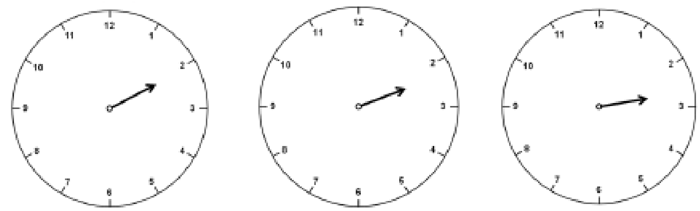
a. Tell and write time in hours and half-hours using analog and digital clocks.

b. Identify the days of the week, the number of days in a week, and the number of weeks in a month.

(*continued on next page*)

1.MD.3a is intended to work with **1.G.3** in which students partition circles into halves and fourths.

By carefully watching and talking about a clock with only the hour hand first, First Graders notice when the hour hand is directly pointing at a number, or when it is slightly ahead/behind a number. In addition, using language such as, "about 5 o'clock," "a little bit past 6 o'clock," and "almost 8 o'clock" helps children begin to read an hour clock more precisely. First Grade students should be able to interpret analog (numbers and hands) and digital clocks, orally tell the time, and write the time to the hour and half-hour using digits.



All of these clocks indicate the hour of "two," although they look slightly different. In the 2nd clock, the hour hand is a little past the 2, showing that we are in the early stages of the 2:00 hour. In the 3rd clock, the hour hand is closer to 3, so you might say, "A little before 3:00"; however, the hour hand is not yet on or past the 3, so the actual time itself would be 2 hours and so many minutes.

<u>Scaffolded approaches for helping students with time</u>*:

- Begin with a one-handed clock (break the minute hand off of a cheap clock) and use approximate language: "about one o'clock," "a little past three o'clock."
- Working with a two-handed clock, discuss the position of the minute hand as the hour hand moves from one number to the next. (Ex: When the hour hand is about halfway between two numbers, where would the minute hand be? When the hour hand is right before a number, about where should the minute hand be?)
- Using two clocks (one with two hands, one with only the hour hand), cover the two-handed clock. Throughout the day, ask students to look at the one-handed clock and predict where the minute hand should be. Then uncover the two-handed clock, check, and discuss.
- Help students make connections between the clock hands and the time they represent. Guide students beyond predicting, "The minute hand should be at 6" to, "It's about 30 minutes after 3:00." A helpful strategy is to focus on the hour first to determine the hour and then the minute hand for more precision.
- Work with analog and digital clocks by covering one and then asking students to predict "about what time" should be on that clock, given what time is showing on the other clock.

- (Van De Walle, *Elementary and Middle School Mathematics: Teaching Developmentally*, 2007)

1.MD.3 (*cont'd*) a. Tell and write time in hours and half-hours using analog and digital clocks. b. Identify the days of the week, the number of days in a week, and the number of weeks in a month.	New Standard (2016 – 17): **1.MD.3b** – The intent of this Standard is to help students establish general calendar knowledge. The most challenging content for students to understand will likely be that there are, essentially, four weeks in one month. Numerically, this is true (28-31 days in a month and 7 days in a week produce 4 weeks *by count* in one month.) However, students should not be expected to do this type of formal division in Grade 1. Students at this age will also likely think of "one week" as "Sunday through Saturday," not the more abstract "span of 7 days" that can begin/end anywhere in the calendar. Teachers should be aware that children who are strongly visual learners or strongly literal thinkers may be able to *say*, "There are seven days in a week. There are four weeks in a month"; but (as described above), they may not *understand* how these things are true. Rather than focus on encouraging students to memorize calendar "facts" without understanding, teachers should encourage familiarity with the calendar, focusing on how it is organized and how we use the calendar to communicate about "when" events occur.
Cluster	
Represent and interpret data.	
Vocabulary: data, category, more, less, most, fewest, same, how many, how many more, how many less	
1.MD.4 Organize, represent, and interpret data with up to three categories; ask and answer questions about the total number of data points, how many in each category, and how many more or less are in one category than in another.	First Grade students collect and use categorical data (e.g., eye color, favorite ice cream flavor) to answer a question. The data collected are often organized in a chart or table. Once the data are collected, First Graders interpret the data to determine the answer to the question posed. They also describe the data noting particular aspects such as the total number of answers, which category had the most/fewest responses, and interesting differences/similarities between the categories. As the teacher provides numerous opportunities for students to create questions, determine up to 3 categories of possible responses, collect data, organize data, and interpret the results, First Graders build a solid foundation for future data representations (picture graphs and bar graphs) in Grade 2 (**2.MD.10**). The purpose of helping the children to determine up to 3 possible responses to a good question before collecting responses is to help young children predict reasonable responses to a question and also to help them organize those responses. Asking, "What is your favorite ice cream flavor?" could provide 10-20 different responses, which might be difficult for them to organize at this stage; and it would also be difficult for them to compare those responses meaningfully. But "Which of these is your favorite ice cream flavor: Vanilla, Orange Sherbet, or Chocolate Chip?" provides a structured question that lends itself better to the early learning stages of data collection, organization, and comparison. This Standard is designed to work well with the "Compare" addition/subtraction problems introduced in First Grade. (See **1.OA.1**, as well as Table 1 at the end of this document.) *Note: Categorical data describes information that belongs to a set of "categories" (ex: eye color, yes/no questions, favorite animals). Categorical data is not something typically represented by a numerical value that can be counted or measured (such as age, height, or number of siblings). Other than the comparisons described above, it is typically not appropriate to perform calculations on categorical data. (Ex: It wouldn't make sense to try to "find the average" of the class's favorite ice cream flavors.)

Cluster	
Work with money.	
Vocabulary: penny, nickel, dime, quarter, half-dollar, dollar, cent, more, less, equal	

1.MD.5
 	
a. Identify the value of all U.S. coins (penny, nickel, dime, quarter, half-dollar, and dollar coins). Use appropriate cent and dollar notation (e.g., 25¢, $1).
 	
b. Know the comparative values of all U.S. coins (e.g., a dime is of greater value than a nickel).
 	
c. Count like U.S. coins up to the equivalent of a dollar
 	
d. Find the equivalent value for all greater value U.S. coins using like value smaller coins (e.g., 5 pennies equal 1 nickel; 10 pennies equal 1 dime, but not 1 nickel and 5 pennies equal 1 dime).
	Standard **1.MD.5** is new for the 2016-17 academic year. Its purpose is to introduce coins and initial concepts about money to students before they solve problems with money in Grade 2. Teachers should be aware that
	• "Skip counting" is not introduced in the Standards until Grade 2 (**2.NBT.2**). Teachers should not expect students to know how to skip count at this stage.
	• Decimal notation is not introduced in the Standards until Grade 4 (**4.NF.6**). Discussions of money in Grade 1 should focus <u>only on whole number amounts</u> with appropriate cent (¢) or dollar ($) notation.
	
	Research has shown several stumbling blocks we can anticipate as children learn about money:
	• Learning to identify the names/characteristics of the coins
	  • Although some coins are bigger in size than others, students often struggle to distinguish between those that are the same color and relatively close in size (ex: nickel and quarter).
	  • Students may also struggle to feel the difference between "rough edges" (ex: dime and quarter) and "smooth edges" (ex: penny and nickel). **Teachers should be aware that many sets of "play money" have smooth edges for all coins, regardless of their real life counterparts. Please check materials before using them in the classroom.**
	• Learning the "worth" of the coins
	  • Our money is an abstract representation.
	    Ex: A nickel is "worth" 5 cents. A student can't *see* the 5 cents in the nickel; there is nothing there to count, physically – there is only 1 coin, and so the student has to associate that with a quantity of "five cents." Research shows that this type of abstract thinking is challenging for young students but is to be expected as part of the learning progression.
	  • Our money system is not proportional. Unlike the Base 10 Blocks which are built to represent the concept that "ten ones (ten Base 10 units) are equal to one ten" (one Base 10 long) and "ten tens (ten Base 10 longs) are equal to one hundred (one Base 10 flat), our money system is not designed the same way.
	    Ex: A dime is "worth" more than a nickel, but a nickel is physically bigger than a dime. Research shows that it is normal for students to struggle with this reasoning, but it is a convention of our money system that we have to help them accept.
	
	This standard may also be used with **1.OA.7** in helping students grasp the meaning of the equal sign as representing "balance" or "the same amount" – as in, "1 dime = 10 pennies."

Geometry

Cluster

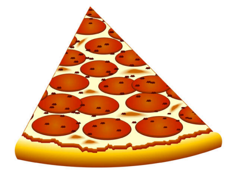
Reason with shapes and their attributes.

Vocabulary: defining attributes, non-defining attributes, closed figure, open figure, two-dimensional (flat), three-dimensional, rectangle, square, trapezoid, triangle, half-circle, quarter-circle, cube, rectangular prism, cone, cylinder, partition, equal shares, halves/half of, fourths/fourth of, quarters/quarter of, whole

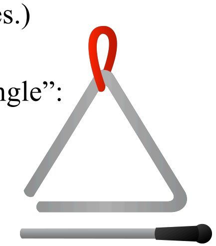
1.G.1
Distinguish between defining attributes (e.g., triangles are closed and three-sided) versus non-defining attributes (e.g., color, orientation, overall size); build and draw shapes to possess defining attributes.

(*continued on next page*)

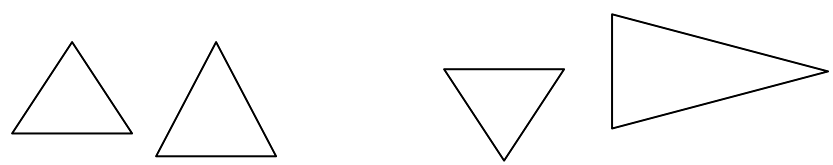
<u>TEACHER NOTE</u>: It is important that teachers be thoughtful in their selection of "real world" examples of shapes. A typical pizza slice is not a good example of a triangle because it has a curved side; triangles have three *straight* sides. Students may initially make similar claims because, in general, "it looks like a triangle." A way to help them address this misunderstanding in a positive way is to clarify, "It's *almost* a triangle. What could we change to make the pizza slice a triangle?" (Make it have three straight sides.)



There is also some natural confusion at this age about the musical instrument called a "triangle": The instrument is called a "triangle" because it obviously resembles a triangle. *However*, you will notice that there is a gap between two of the three sides. This does not fit the mathematical definition of a triangle: Triangles have three straight sides and are "closed" (all sides touching with no gaps between them). But this "conflict" can be easily resolved with young children with similar conversations to the pizza slice above: "Musicians call this instrument a triangle because it looks a lot like a triangle. But we know it's not a 'math triangle.' What could we change to make it a math triangle?" (Make sure that all sides touch and that the sides are straight, not curved.)



Students should have experience with shapes from Kindergarten. However, some students' picture of a shape is often limited to an "ideal" version of the shape. For example, many young students will identify an equilateral triangle or isosceles triangle as a triangle because "it looks like" the triangles they have seen. However, they may also claim that triangles with different orientations or elongated sides are *not* triangles because "they don't look like triangles":



Triangles   "Not Triangles"

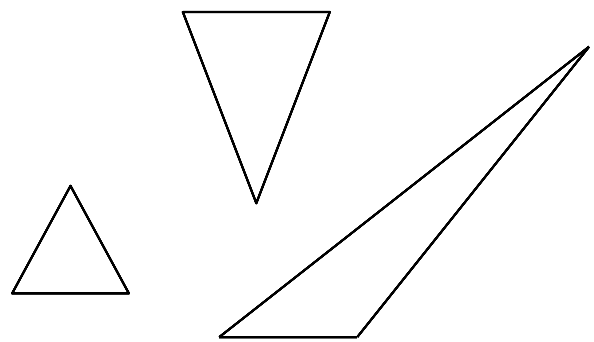
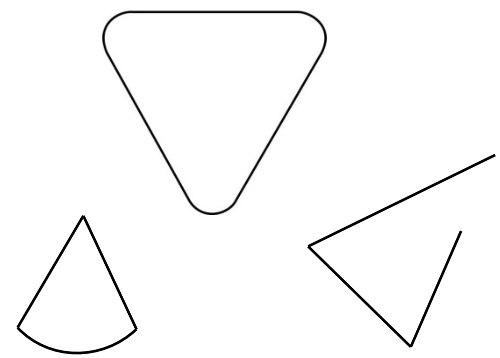
Young Children's Initial Claims, Based on What the Shapes "Look Like"

1.G.1 (*cont'd*)
Distinguish between defining attributes (e.g., triangles are closed and three-sided) versus non-defining attributes (e.g., color, orientation, overall size); build and draw shapes to possess defining attributes.

(*continued on next page*)

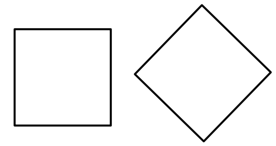
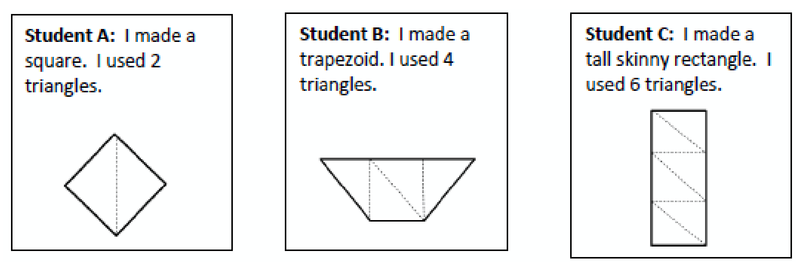
Classroom conversations can help resolve these misconceptions. Teachers can facilitate age-appropriate discussions about "What 'makes' this shape a triangle?" and guide students through attention to specific features (ex: three straight sides that touch) versus general, yet ultimately inaccurate ideas of "pointy" or "flat on the bottom."

Research also shows that presenting young students with examples and non-examples (see below) and facilitating discussions of "What do you notice?... What is 'the same' about the triangles? Why are the other shapes not with the triangles?" can help them learn how to pay attention to defining features of shapes (ex: number of sides, straight sides, etc.) rather than non-defining features (ex: orientation, size, color).

These are triangles	These are not triangles
	

<u>TEACHER NOTE</u>: The mathematical attributes of a rectangle *do not include* "having two long sides and two short sides." Those characteristics <u>should not</u> be taught as defining attributes of a rectangle. In its most general terms, a rectangle has four straight sides and four "square corners" (or right angles). The opposite sides of a rectangle are the same length and are parallel. (If you extended the sides, they would never touch.) A square fits all of the characteristics of a rectangle. A square is a *special type of rectangle* in that <u>all</u> sides of a square are the same length.

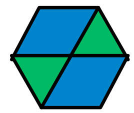
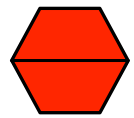
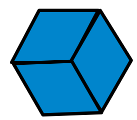
A developmentally appropriate understanding of the relationship between squares and rectangles could be that "squares are part of the rectangle family."

1.G.1 (*cont'd*) Distinguish between defining attributes (e.g., triangles are closed and three-sided) versus non-defining attributes (e.g., color, orientation, overall size); build and draw shapes to possess defining attributes.	It is also important to note that orientation of a figure does not change the figure itself. Given the shapes below, students often refer to the figure on the left as a square and the figure on the right as a diamond. Both figures are squares; the square on the right has just been rotated. (This connects back to **K.G.2**.)  Unfortunately, many "educational" materials refer to a rhombus, or even a rotated square, as a "diamond." "Diamond" is not a geometric term and should not be used to describe shapes. ***TEACHER NOTE***: In the U.S., the term "trapezoid" may have two different meanings. Research identifies these as inclusive and exclusive definitions. The inclusive definition states: A trapezoid is a quadrilateral with at least one pair of parallel sides. With this definition, parallelograms, rectangles, squares, and rhombi fit that definition and can thus be considered as *types of trapezoids*. The exclusive definition states: A trapezoid is a quadrilateral with exactly one pair of parallel sides. With this definition, parallelograms and their subgroups do not fit the definition and thus are not considered to be types of trapezoids. (*Progressions for the CCSSM: Geometry*, The Common Core Standards Writing Team, June 2012) The following articles (written for teachers) offer helpful advice for discussing shapes with young students: • Hannibal, M. A. (1999, February). Young children's developing understanding of geometric shapes. *Teaching Children Mathematics*, 353-357. • Roberts, S. K. (2007, December). Watch what you say. *Teaching Children Mathematics*, 296-301.
1.G.2 Compose two-dimensional shapes (rectangles, squares, trapezoids, triangles, half-circles, and quarter-circles) or three-dimensional shapes (cubes, right rectangular prisms, right circular cones, and right circular cylinders) to create a composite shape, and compose new shapes from the composite shape.[4] [4] Students do not need to learn formal names such as "right rectangular prism."	This Standard builds off of students' work with composing and decomposing shapes in Kindergarten (**K.G.6**). Example: What shapes can you create with triangles? 

1.G.2 (*cont'd*)
Compose two-dimensional shapes (rectangles, squares, trapezoids, triangles, half-circles, and quarter-circles) or three-dimensional shapes (cubes, right rectangular shapes (cubes, right rectangular prisms, right circular cones, and right circular cylinders) to create a composite shape, and compose new shapes from the composite shape.[4]

[4] Students do not need to learn formal names such as "right rectangular prism."

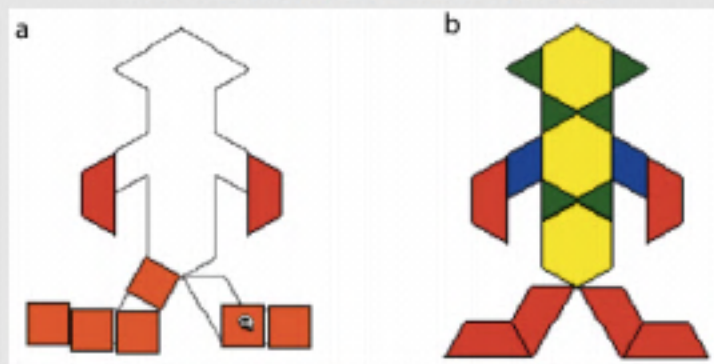
<u>Example</u>: How many ways can you use Pattern Blocks to make the same size and shape as the yellow hexagon? How many of each shape did you use? Use the paper shapes and glue stick to make

What You Built	How Many Shapes You Used
	*2 rhombi and 2 triangles*
	*2 trapezoids*
	*3 rhombi*

(Note: There are more possibilities than those contained in the chart above.)

Students might also be given "puzzles" and asked to use Pattern Blocks to complete the figure:

Combining shapes to build pictures



Students first use trial and error (part a) and gradually consider components (part b).

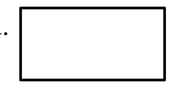
1.G.3
Partition circles and rectangles into two and four equal shares, describe the shares using the words *halves, fourths,* and *quarters*, and use the phrases *half of, fourth of, and quarter of.* Describe the whole as two of, or four of the shares. Understand for these examples that decomposing into more equal shares creates smaller shares.

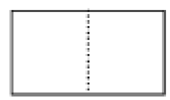
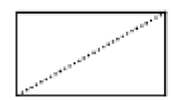
The focus in Grade 1 is using *names/words* (ex: one half) to describe fractions, not symbols (ex: $\frac{1}{2}$).

Research shows that "repeated halving" (cutting a whole in half, then cutting those halves in half, etc.) is a powerful strategy for helping students learn how to split one whole into smaller equal parts. The Standards incorporate this research to scaffold students' work with fractions across Grades 1- 4:

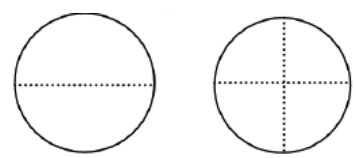
Grade	**Expectations**
1st Grade	Halves and fourths (words/names, not symbols/fraction notation)
2nd Grade	Halves, thirds, and fourths (words/names, not symbols/fraction notation)
3rd Grade	Denominators of 2, 4, 8, 3 and 6 (words *and* symbols/fraction notation)
4th Grade	Denominators of 2, 4, 8, 16, 3, 6, 12, 5, 10, and 100 (words *and* symbols/fraction notation)

Example: Suppose this is a brownie that you want to share with a friend. How can you cut the brownie so that you both get a fair share?



Student 1	**Student 2**
I would cut the brownie in the middle to make two equal parts. So, I get one half, and my friend would get one half. 	I would cut it from the top right to the bottom left. Those triangle pieces are the same size, so we'd both get the same amount. If he didn't believe me, we could stack them on top of each other to see that they're the same. 

Example: There is pizza for dinner. Would you rather have a piece of pizza from the one on the left or from the one on the right?



Student: The pizzas are the same size; but the more parts that you cut the pizza into, the smaller the parts are. The one on the left has two halves, and the one on the right has four fourths. Halves are bigger than fourths, so I'd rather have a piece from the pizza on the left.

Table 1: Common Addition and Subtraction Situations

	Result Unknown	Change Unknown	Start Unknown
Add To	Two bunnies sat on the grass. Three more bunnies hopped there. How many bunnies are on the grass now? $2 + 3 = ?$ (K)	Two bunnies were sitting on the grass. Some more bunnies hopped there. Then there were five bunnies. How many bunnies hopped over to the first two? $2 + ? = 5$ (1^{st})	Some bunnies were sitting on the grass. Three more bunnies hopped there. Then there were five bunnies. How many bunnies were on the grass before? $? + 3 = 5$ One-Step Problem (2^{nd})
Take From	Five apples were on the table. I ate two apples. How many apples are on the table now? $5 – 2 = ?$ (K)	Five apples were on the table. I ate some apples. Then there were three apples. How many apples did I eat? $5 – ? = 3$ (1^{st})	Some apples were on the table. I ate two apples. Then there were three apples. How many apples were on the table before? $? – 2 = 3$ One-Step Problem (2^{nd})
	Total Unknown	**Addend Unknown**	**Both Addends Unknown**
Put Together/Take Apart	Three red apples and two green apples are on the table. How many apples are on the table? $3 + 2 = ?$ (K)	Five apples are on the table. Three are red and the rest are green. How many apples are green? $3 + ? = 5$ or $5 – 3 = ?$ (1^{st})	Grandma has five flowers. How many can she put in her red vase and how many in her blue vase? $5 = 0 + 5$, $5 = 5 + 0$ $5 = 1 + 4$, $5 = 4 + 1$ $5 = 2 + 3$, $5 = 3 + 2$ (K)
	Difference Unknown	**Bigger Unknown**	**Smaller Unknown**
Compare	(“How many more?” version): Lucy has two apples. Julie has five apples. How many more apples does Julie have than Lucy? (1^{st})	(Version with “more”): Julie has three more apples than Lucy. Lucy has two apples. How many apples does Julie have? One-Step Problem (1^{st})	(Version with “more”): Julie has 3 more apples than Lucy. Julie has five apples. How many apples does Lucy have? $5 – 3 = ?$ or $? + 3 = 5$ One-Step Problem (2^{nd})
Compare	(“How many fewer?” version): Lucy has two apples. Julie has five apples. How many fewer apples does Lucy have than Julie? $2 + ? = 5$ or $5 – 2 = ?$ (1^{st})	(Version with “fewer”): Lucy has 3 fewer apples than Julie. Lucy has two apples. How many apples does Julie have? $2 + 3 = ?$ or $3 + 2 = ?$ One-Step Problem (2^{nd})	(Version with “fewer”): Lucy has 3 fewer apples than Julie. Julie has five apples. How many apples does Lucy have? $5 – 3 = ?$, $? + 3 = 5$ One-Step Problem (1^{st})

K: Problem types to be mastered by the end of the Kindergarten year. **1^{st}**: Problem types to be mastered by the end of the First Grade year, including problem types from the previous year. However, First Grade students should have experiences with all 12 problem types. **2^{nd}**: Problem types to be mastered by the end of the Second Grade year, including problem types from the previous years.

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